

III Semester B.A./B.Sc. Examination, Nov./Dec. 2017
(NS) (2012-13 and Onwards) (Repeaters - Prior to 2015-16)
MATHEMATICS - III

Time : 3 Hours

Max. Marks : 100

Instruction : Answer all questions.

I. Answer any fifteen questions :

(15×2=30)

- 1) Define normal subgroup.
- 2) Show that every quotient group of an abelian group is abelian.
- 3) If $f : G \rightarrow G'$ be a homomorphism from the group (G, \cdot) into a group $(G', *)$ then prove that $f(a^{-1}) = [f(a)]^{-1} \forall a \in G$.

4) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ find $f^{-1}og$.

- 5) What is linear programming problem ?
- 6) Solve $3x + 2y > 6$ graphically, $x \geq 0, y \geq 0$.
- 7) Using North West corner method determine an initial basic solution of the following transportation problem.

	1	2	3	Available
A	15	26	13	340
B	3	7	8	100
C	9	4	3	110
Requirement	80	150	320	

- 8) Define convergent and divergent sequences.
- 9) Test the convergence of the sequence $\sqrt{n+1} - \sqrt{n}$.
- 10) Show that $\left\{ \frac{n}{n+1} \right\}$ is a Cauchy sequence.

11) Prove that the sequence $\{x_n\}$ whose n^{th} term is $\frac{n+3}{n+4}$ is monotonic increasing



- 13) Define Geometric series.
- 14) Test the convergence of the series $\sum \sin\left(\frac{1}{n}\right)$.
- 15) Test the convergence of the series $\sum \frac{n^2}{n!}$.
- 16) Sum to infinity of the series $1 + \frac{x}{3} + \frac{x^2}{5} + \frac{x^3}{7} + \dots$
- 17) Write the Geometrical interpretation of Rolle's theorem.
- 18) Prove that the function $f(x) = \begin{cases} \frac{|x|}{x} & \text{for } x \neq 0 \\ 1 & \text{for } x = 0 \end{cases}$ is discontinuous at $x = 0$.
- 19) Expand $\cos x$ by Maclaurin's expansion.
- 20) Evaluate $\lim_{x \rightarrow 0} \left(\frac{x - \sin x}{x^3} \right)$.

II. Answer **any two** questions :

(2x5=10)

- 1) Prove that a subgroup H of a group G is normal if and only if $gHg^{-1} = H \quad \forall g \in G$.
- 2) Prove that the centre Z of a group G is a normal subgroup of G .
- 3) If $f : G \rightarrow G'$ is a homomorphism from the group G into the group G' with Kernel K , then prove that f is one-one if and only if $K = \{e\}$.
- 4) State and prove Cayley's theorem.

III. Answer **any three** questions :

(3x5=15)

- 1) Find all the basic solutions of the system of equation $x + 2y + z = 4$ and $2x + y + 5z = 5$.
- 2) Solve the following L.P.P. graphically
 Minimize, $z = -x + 2y$
 Subject to constraints $-x + 3y \leq 10$
 $x + y \leq 6$
 $x - y \leq 2$
 and $x > 0, y > 0$



3) Solve the following L.P.P. by Simplex method

Maximize, $z = 4x + 3y$

Subject to constraints $3x + y \leq 15$

$3x + 4y \leq 24$

and $x, y, \geq 0.$

4) Obtain an initial basic solution to the following transportation problem using Vogel's method.

	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	19	30	50	10	7
S ₂	70	30	40	60	9
S ₃	40	8	70	20	18
Demand	5	8	7	14	

IV. Answer any two questions :

(2x5=10)

1) Prove that a monotonic increasing sequence bounded above is convergent.

2) Discuss the nature of the sequence $\{n^{1/n}\}$.

3) Show that the sequence $\{x_n\}$ where $x_1 = 1$ and $x_n = \sqrt{2 + x_{n-1}} \forall n > 1$, is convergent and converges to 2.

(4x5=20)

V. Answer any four questions :

1) Prove that P-series $\sum \frac{1}{n^p}$ is

i) Convergent if $P > 1$ and

ii) Divergent if $P \leq 1.$

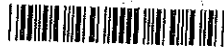
2) State and prove Raabe's test.

3) Test the convergence of the series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots$

4) Discuss the convergence of the series $1 + 2^2 \cdot x + 3^2 \cdot x^2 + 4^2 \cdot x^3 + \dots$

5) Sum to infinity the series $\sum_{n=1}^{\infty} \left(\frac{n^2 + n + 1}{n!} \right) x^n.$

6) Sum to infinity the series $\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$



VI. Answer **any three** questions :

(3x5=15)

1) Examine the differentiability of $f(x)$ defined by

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x \geq 1 \\ 1 - x & \text{for } x < 1 \end{cases} \text{ at } x = 1.$$

2) State and prove Rolle's theorem.

3) Verify Lagrange's mean value theorem for $f(x) = (x-1)(x-2)(x-3)$ in $[0, 4]$.

4) Expand the function $\log_e(1+x)$ upto the term containing x^4 by Maclaurin's expansion.

5) Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \cot^2 x \right)$.

BMSCW
