SN – 35

III Semester B.A./B.Sc. Examination, Nov./Dec. 2017
(NS) (2012-13 and Onwards) (Repeaters – Prior to 2015-16)
MATHEMATICS – III

Time : 3 Hours Max. Marks : 10

Instruction : *Answer **all** questions.*

(15×2=3

I. Answer **any fifteen** questions :

1) Define normal subgroup.

2) Show that every quotient group of an abelian group is abelian.

3) If $f : G \to G'$ be a homomorphism from the group $(G, \bullet)$ into a group $(G', *)$ then prove that $f(a^{-1}) = [f(a)]^{-1}\ \forall a \in G$.

4) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ find $f^{-1}og$.

5) What is linear programming problem ?

6) Solve $3x + 2y > 6$ graphically, $x \geq 0$, $y \geq 0$.

7) Using North West corner method determine an initial basic solution of the following transportation problem.

	1	2	3	Available
A	15	26	13	340
B	3	7	8	100
C	9	4	3	110
Requirement	80	150	320	

8) Define convergent and divergent sequences.

9) Test the convergence of the sequence $\sqrt{n+1} - \sqrt{n}$.

10) Show that $\left\{\frac{n}{n+1}\right\}$ is a Cauchy sequence.

11) Prove that the sequence $\{x_n\}$ whose n^{th} term is $\frac{n+3}{n+4}$ is monotonic increasing

... prove that $\lim u_n = 0$

13) Define Geometric series.

14) Test the convergence of the series $\Sigma \sin\left(\frac{1}{n}\right)$.

15) Test the convergence of the series $\Sigma \frac{n^2}{n!}$.

16) Sum to infinity of the series $1+\frac{x}{3}+\frac{x^2}{5}+\frac{x^3}{7}+\ldots$

17) Write the Geometrical interpretation of Rolle's theorem.

18) Prove that the function $f(x) = \begin{cases} \frac{[x]}{x} & \text{for } x \neq 0 \\ 1 & \text{for } x = 0 \end{cases}$ is discontinuous at x =0.

19) Expand cosx by Maclaurin's expansion.

20) Evaluate $\lim_{x \to 0}\left(\frac{x - \sin x}{x^3}\right)$.

II. Answer **any two** questions : **(2×5=10)**

1) Prove that a subgroup H of a group G is normal if and only if $gHg^{-1} = H \ \forall g \in G$.

2) Prove that the centre Z of a group G is a normal subgroup of G.

3) If $f : G \to G'$ is a homomorphism from the group G into the group G' with Kernel K, then prove that f is one-one if and only if $K = \{e\}$.

4) State and prove Cayley's theorem.

III. Answer **any three** questions : **(3×5=15)**

1) Find all the basic solutions of the system of equation $x + 2y + z = 4$ and $2x + y + 5z = 5$.

2) Solve the following L.P.P. graphically

Minimize, $z = -x + 2y$

Subject to constraints $-x + 3y \leq 10$

$x + y \leq 6$

$x - y \leq 2$

and $x \geq 0, y \geq 0$

3) Solve the following L.P.P. by Simplex method

Maximize, $z = 4x + 3y$

Subject to constraints $3x + y \leq 15$

$3x + 4y \leq 24$

and $x, y, \geq 0$.

4) Obtain an initial basic solution to the following transportation problem using Vogel's method.

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	

(2x5=1

IV. Answer **any two** questions :

1) Prove that a monotonic increasing sequence bounded above is convergent.

2) Discuss the nature of the sequence $\left\{n^{1/n}\right\}$.

3) Show that the sequence $\{x_n\}$ where $x_1 = 1$ and $x_n = \sqrt{2 + x_{n-1}}$ $\forall n > 1$, is convergent and converges to 2.

(4x5=2

V. Answer **any four** questions :

1) Prove that P-series $\Sigma \frac{1}{n^p}$ is

i) Convergent if $P > 1$ and   ii) Divergent if $P \leq 1$.

2) State and prove Raabe's test.

3) Test the convergence of the series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots..$

4) Discuss the convergence of the series $1 + 2^2 \cdot x + 3^2 \cdot x^2 + 4^2 \cdot x^3 + \ldots.$

5) Sum to infinity the series $\sum_{n=1}^{\infty} \left(\frac{n^2 + n + 1}{n!}\right) x^n$.

6) Sum to infinity the series $\frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4} - \frac{1}{4.5} + \ldots..$

VI. Answer **any three** questions : **(3×5=15)**

1) Examine the differentiability of f(x) defined by

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x \geq 1 \\ 1 - x & \text{for } x < 1 \end{cases} \text{ at } x = 1.$$

2) State and prove Rolle's theorem.

3) Verify Lagrange's mean value theorem for $f(x) = (x-1)(x-2)(x-3)$ in [0,4].

4) Expand the function $\log_e(1 + x)$ upto the term containing x^4 by Maclaurin's expansion.

5) Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \cot^2 x \right)$.
